

# ISI Type-B Mock Test

Answer as much as you can. Best of Luck!

1. Prove the following inequality for  $a \geq b > 0$ .

$$\frac{(a-b)^2}{8a} \leq \frac{a+b}{2} - \sqrt{ab} \leq \frac{(a-b)^2}{8b}$$

**(8 marks)**

2. Find all real number  $x$  such that  $10^x + 11^x + 12^x = 13^x + 14^x$ . **(8 marks)**

3. Prove that for any  $n \geq 6$ , the equation

$$\frac{1}{x_1^2} + \frac{1}{x_2^2} + \dots + \frac{1}{x_n^2} = 1$$

has an integral solutions in  $x_1, x_2, \dots, x_n$ . **(12 marks)**

4. Let  $AB$  be a chord of a circle. Let  $P$  be a variable point on the arc  $AB$ . Show that,  $AP + BP$  is maximum when  $P$  is the midpoint of the arc  $AB$ . (This statement is very obvious, but I need you to write a concise properly written mathematical proof of the fact) **(10 marks)**

5. The sum of the digits in the decimal representation of  $4444^{4444}$  is  $A$ . Let the sum of digits of  $A$  is  $B$ . What is the sum of the digits in  $B$ ? **(12 marks)**

6. Consider the following;

1. Which one is bigger among these  $2^{1/2}, 3^{1/3}, 6^{1/6}$ ? **(4 marks)**
2. Which one is bigger among  $e^\pi$  and  $\pi^e$ ? **(6 marks)**