

ISI Type-B Mock Test

Answer as much as you can. Best of Luck!
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1. Prove the following inequality for $a \geq b > 0$.

$$\frac{(a-b)^2}{8a} \leq \frac{a+b}{2} - \sqrt{ab} \leq \frac{(a-b)^2}{8b}$$

(8 marks)

2. Find all real number x such that $10^x + 11^x + 12^x = 13^x + 14^x$. **(8 marks)**

3. Prove that for any $n \geq 6$, the equation

$$\frac{1}{x_1^2} + \frac{1}{x_2^2} + \dots + \frac{1}{x_n^2} = 1$$

has an integral solutions in x_1, x_2, \dots, x_n . **(12 marks)**

4. Let AB be a chord of a circle. Let P be a variable point on the arc AB . Show that, $AP + BP$ is maximum when P is the midpoint of the arc AB . (This statement is very obvious, but I need you to write a concise properly written mathematical proof of the fact) **(10 marks)**

5. The sum of the digits in the decimal representation of 4444^{4444} is A . Let the sum of digits of A is B . What is the sum of the digits in B ? **(12 marks)**

6. Consider the following;

1. Which one is bigger among these $2^{1/2}, 3^{1/3}, 6^{1/6}$? **(4 marks)**
2. Which one is bigger among e^π and π^e ? **(6 marks)**